**December 1**

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Q4

The slope of a graph of entropy vs. energy (dS/dE) in a metal block is related to the temperature of the block. From the graph of entropy for two blocks in contact, we see that the block with the larger slope tends to gain energy from the block with the smaller slope. Therefore, which of these statements is true?

A) Big dS/dE means high temperature
B) Small dS/dE means high temperature

$$\frac{dS}{dE} = \frac{1}{T}$$

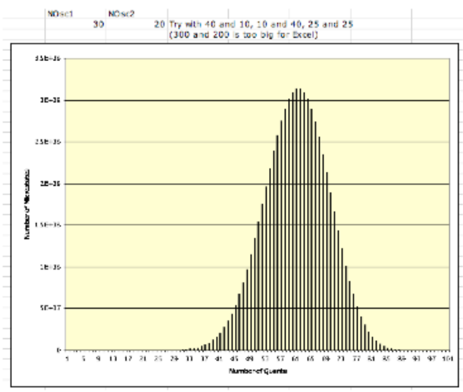
Q5

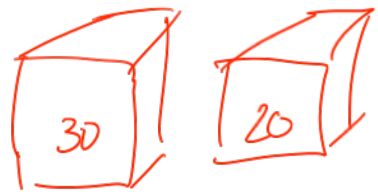
| | |
|---|---|
| There is thermal transfer of energy of 5000 J into a system. The entropy of the system increases by 50 J/K.<br><br>What is the approximate temperature of the system? | A) 5000 K<br>B) 100 K<br>C) 50 K<br>D) 0.01 K<br>E) 0.0002 K |

Q6

| | |
|---|---|
| Consider a 3 kg block of aluminum. One mole of aluminum has a mass of 27 grams (0.027 kg). From Young's modulus we determined that the stiffness of the interatomic bond is 16 N/m, but in the Einstein model the *x*, *y*, and *z* oscillations each involve 2 half-length springs, so the effective stiffness is 64 N/m. $\hbar = 1.05\times10^{-34}\ \text{J}\cdot\text{s}$; $1\ \text{mol} = 6\times10^{23}$ atoms | What is the energy in joules of one quantum of energy?<br>A) 1.62e-34 J<br>B) 4.85e-34 J<br>C) 5.11e-33 J<br>D) 1.25e-22 J<br>E) 3.96e-21 J |

$$E = \hbar\omega = \hbar\sqrt{\frac{k_s}{m}} = \hbar\sqrt{\frac{k_s}{M/N_A}}$$




30 oscillators in block 1, 20 oscillators in block 2
100 quanta

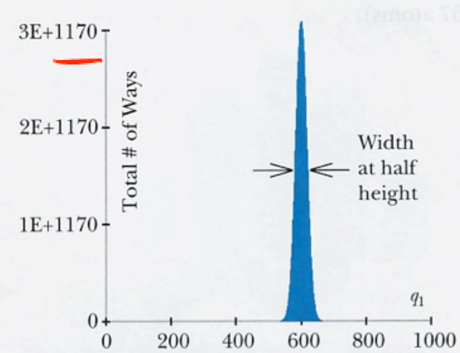
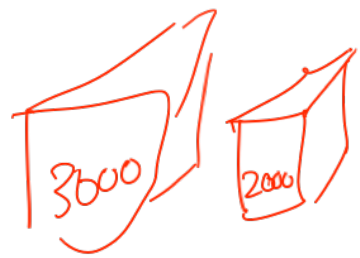



**Figure 11.21** The number of ways of distributing 1000 quanta between two blocks containing 3000 and 2000 oscillators, respectively.



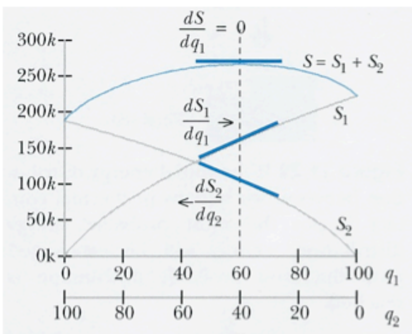



**Figure 11.27** Entropy *vs.* number of quanta of energy in system 1.

$$S = k \ln \Omega$$

\# of ways to arrange

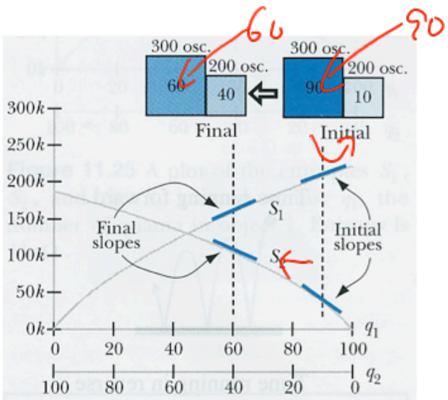



**Figure 11.28** Location of the initial and final states of the two-block system on a plot of entropy *vs.* number of energy quanta in system 1.

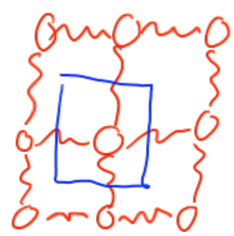
$$\frac{dS}{dE} = \frac{1}{T}$$

## What's this thing called quanta?

Consider a 3 kg block of aluminum. One mole of aluminum has a mass of 27 grams (0.027 kg). From Young's modulus we determined that the stiffness of the interatomic bond is 16 N/m, but in the Einstein model the *x*, *y*, and *z* oscillations each involve 2 half-length springs, so the effective stiffness is 64 N/m. What is the energy in joules of one quantum of energy?

$\hbar = 1.05 \times 10^{-34}\ \mathrm{J \cdot s}$

$4 \times 16\ \mathrm{N/m} = 64\ \mathrm{N/m}$



$E = \hbar\omega = 3.97 \times 10^{-21}\ \mathrm{J}$

$= \hbar \sqrt{\frac{k_s}{m_a}}$

$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$

1 quantum at a time

$\Delta E = 3.97 \times 10^{-21}\ \mathrm{J}$

100 atoms

k = 1.38E-23 hbar = 1.05E-34

Aluminum ks = 64 Mmole = 0.027

one quantum = 3.98446E-21 J

| q | omega | E | S | ΔE | ΔS | T |
|---|---|---|---|---|---|---|
| 20 | 4.9100E+26 | 7.9689E-20 | 8.4856E-22 | | | |
| | | | | 3.9845E-21 | 3.0403E-23 | 131.06 |
| 21 | 4.4400E+27 | 8.3674E-20 | 8.7896E-22 | | | |
| | | | | 3.9845E-21 | 2.9823E-23 | 133.60 |
| 22 | 3.8500E+28 | 8.7658E-20 | 9.0878E-22 | | | |

heat cap/atom 1.5650E-23 from mid-point to mid-point

$$\frac{1}{T} = \frac{dS}{dE}$$

$$T = \frac{\Delta E}{\Delta S}$$

heat Capacity

$$C = \frac{dE}{dT}$$

$$C = \frac{dE}{dT} / \# \text{ atoms}$$

heat capacity/atom

100 atoms

Gold ks = 20 Mmole = 0.197

one quantum = 1.6492E-21 J $= E_1$

| q | omega | E | S | ΔE | ΔS | T |
|---|---|---|---|---|---|---|
| 20 | 4.9100E+26 | $20E_1$ | $k \ln \Omega$ | $E_1$ | $S_{21}-S_{20}$ | $\frac{E_1}{S_{21}-S_{20}} = T_{20.5}$ |
| 21 | 4.4400E+27 | $21E_1$ | " | | | |
| 22 | 3.8500E+28 | $22E_1$ | " | $E_1$ | $S_{22}-S_{21}$ | $E_1/S_{22}-S_{21} = T_{21.5}$ |
| | C=? | | | | | |

$$C = \frac{\frac{E_1}{T_{21.5}-T_{20.5}}}{100}$$

Tungsten ks = 90 Mmole = 0.185

one quantum = 3.61017E-21 J

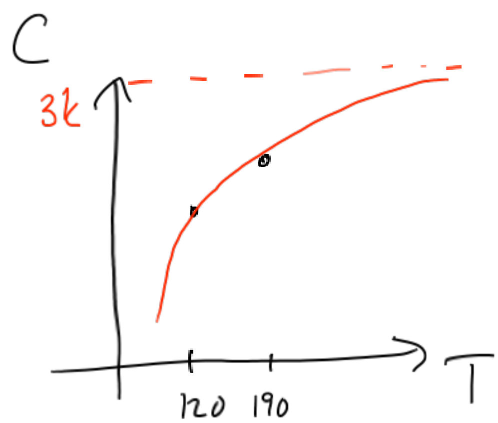
| q | omega | E | S | ΔE | ΔS | T |
|---|---|---|---|---|---|---|
| 20 | 4.9100E+26 | | | | | |
| 21 | 4.4400E+27 | | | | | |
| 22 | 3.8500E+28 | | | | | |
| | C=? | | | | | |

Copper ks = 28 Mmole = 0.064

one quantum = 3.42358E-21 J

| q | omega | E | S | ΔE | ΔS | T |
|---|---|---|---|---|---|---|
| 20 | 4.9100E+26 | | | | | |
| 21 | 4.4400E+27 | | | | | |
| 22 | 3.8500E+28 | | | | | |
| | C=? | | | | | |

$c \approx 1.5 \times 10^{-23}$ J/K




$3k = 4.2 \times 10^{-23}$ J/K per atom

1 k for each independent oscillator

3 for x, y, z oscillators

**Ponderable: Phase Changes**

1 g of ice at 0°C + 335 J yields 1 g of liquid water at same temperature. The entropy increases. Latent heat of fusion $L_f$ = 335 J/g. What is the change in entropy?

$$\frac{1}{T} = \frac{\Delta S}{\Delta E}$$

$$\Rightarrow \Delta S = \frac{\Delta E}{T} = \frac{335\,J}{273\,K} = 1.2\,J/K$$

1 g of liquid water at room temp 20°C + energy yields 1 g of liquid water at 100°C. What is the change in entropy?

$$C = 4.2\ J/gK$$

$$\Delta S = S_{373} - S_{293} \qquad \frac{dS}{dE} = \frac{1}{T}$$

$$dS = \frac{dE}{T} \qquad C = \frac{dE}{dT} \Rightarrow dE = C\,dT$$

$$\Delta S = \int \frac{dE}{T} = C \int_{293}^{373} \frac{dT}{T} = C \ln \frac{373}{293} = 4.2\frac{J}{K} \ln \frac{373}{293}$$

$$= 1.0\ J/K$$

A certain object (for which the Einstein solid is not a good model) has entropy $S = bE^{1/2}$, where b is a constant. We know that $\frac{1}{T} = \frac{dS}{dE}$. Find the temperature T as a function of energy E.

$$\frac{dS}{dE} = \frac{1}{T}$$

$$\frac{dS}{dE} = \frac{1}{2} b E^{-1/2} = \frac{d}{dE}\left(bE^{1/2}\right)$$

$$T = \frac{1}{\frac{1}{2} b E^{-1/2}} = \frac{2}{b} E^{1/2}$$

Suppose the object has a mass of 1 gram. Calculate the heat capacity on a per-gram basis as a function of the temperature T, starting from $T = \frac{2}{b} E^{1/2}$.

$$E = \frac{b^2}{4} T^2$$

$$C = \frac{dE}{dT} = \frac{d}{dT}\left(\frac{b^2}{4} T^2\right)$$

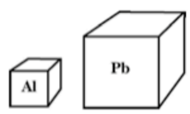
$$= \frac{b^2}{4} \frac{d}{dT}\left(T^2\right) = \frac{b^2}{2} T$$

**Clicker questions**

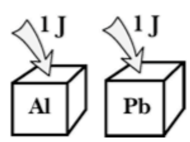
Q1: We have two blocks, one aluminum (Al) and one lead (Pb), each containing 6e23 atoms (one mole). The aluminum block has a mass of 27 grams, and the lead block has a mass of 207 grams. Which of the following pictures shows the blocks in the correct relative sizes?

A) 

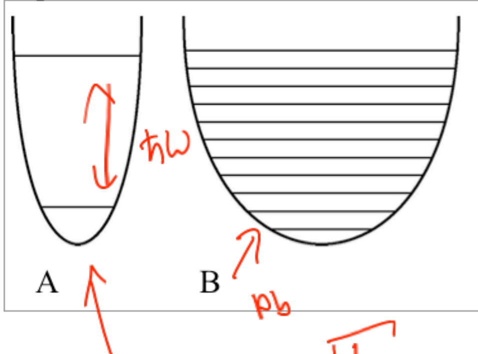

B) 


C) 

Initially the two blocks are at a temperature very near absolute zero (0 K). We will add 1 J of energy to the aluminum block, and 1 J of energy to the lead block, and see which block has the larger increase in temperature. We will step through a chain of reasoning using statistical mechanics to answer this question, which will let us determine whether aluminum or lead has the higher heat capacity at low temperatures.



Q2: From Young's modulus we found that the effective stiffness of the interatomic bond for Al is about 16 N/m and for Pb is about 5 N/m. A mole of Al is 27 grams, and a mole of Pb is 207 grams. Here are energy level diagrams for the quantized harmonic oscillators used in the Einstein solid. Which diagram represents Al and which represents Pb?




A) *(A)* is Al and *(B)* is Pb

B) *(A)* is Pb and *(B)* is Al

C) *(B)* is both Al and Pb (they are the same)

$$\omega = \sqrt{\frac{k_s}{m_a}}$$

$$\omega_{Al} = \sqrt{\frac{4\cdot 16\,N/m}{.027\,kg/N_A}} \gg \omega_{Pb} = \sqrt{\frac{4\cdot 5\,N/m}{.207\,kg/N_A}}$$

Q3: We add 1 J of energy to each block. Given the fact that Al has the greater energy-level spacing, which block now has the larger number of quanta of energy, *q?*

A) The number of quanta *q* is greater in the Al

B) The number of quanta *q* is greater in the Pb

C) The number of quanta *q* is the same for Al and Pb

Q4: What about the number $N$ of quantized oscillators in the two blocks?

A) $N$ is greater in the Al

B) $N$ is greater in the Pb

C) $N$ is the same for Pb and Al

3x # of atoms

Q5: The Pb block has more quanta corresponding to the 1J of thermal energy. Therefore, in which block is there a larger number of ways **Ω** of arranging the thermal energy?

A) The number of ways **Ω** is greater in the Al

B) The number of ways Ω is greater in the Pb

C) The number of ways **Ω** is the same in the Pb and the Al

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$$

Q6: The Pb block has the larger number of ways $\mathbf{\Omega}$ to arrange the energy. So which block now has the larger entropy *S?*

A) The entropy S is now greater in the Al

B) The entropy S is now greater in the Pb

C) The entropy S is the same in the Al and the Pb

$S = k \ln \Omega$

Q7: Originally the temperature of the blocks was near absolute zero, with almost no thermal energy in the blocks. How many ways are there to arrange zero energy in a block? Just 1. So what was the original entropy in a block?

A) 0 J/K

B) 1 J/K

C) infinite

Q8: We found that after adding 1 J to each block, the entropy $S$ is now greater in the Pb block. Both blocks started with zero entropy. Therefore which block experienced a larger *change* in entropy $\Delta S$?

A) The entropy change $\Delta S$ was greater in the Al

B) The entropy change $\Delta S$ was greater in the Pb

C) The entropy change $\Delta S$ was the same in the Pb and the Al

Q9: We added the same amount of energy $\Delta E$ = 1 J to each block, and the entropy change $\Delta S$ was greater in the Pb block. Which block now has the higher temperature?

A) The temperature of the Al is now higher

B) The temperature of the Pb is now higher

C) The temperature of the Al and Pb are the same

$$\frac{1}{T} = \frac{\Delta S}{\Delta E}$$

Q10: The original temperature was 0 K, and the final temperature of the Al block is higher than that of the Pb block, so the Al block has the larger *change* in temperature, $\Delta T$. At low temperatures, which block has the greater heat capacity per atom, $C = (\Delta E/\Delta T)/6e23$?

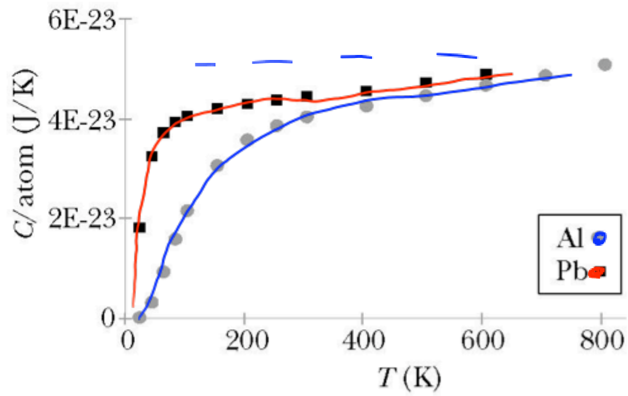
A) The low-temperature heat capacity per atom of Al is greater

B) The low-temperature heat capacity per atom of Pb is greater

C) The low-temperature heat capacity per atom is the same for Pb and Al

$$C = \frac{\Delta E}{\Delta T}$$

Here are actual heat capacity data for Al and Pb (see textbook):




3k Classical limit

Boltzmann's const